Tribhuwan University
Institute of Science and Technology
2078

Bachelor Level / second-semester / Science Full marks: 80 **Computer Science and Information Technology(MTH163)** Pass marks: 32 (Mathematics II) Time: 3 hours Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

Group A

Attempt any THREE questions. (10 x 3 = 30)

1. Define a system of linear equations. When a system of equations is consistent? Determine if the system

$-2x_1-3x_2+4x_3 = 5$

$x_2-2x_3 = 4$

$x_1+3x_2-x_3 = 2$

is consistent. [1+1+8]

2. Define linear transformation with an example. [1+1+3+5]

Let $A = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix}$, $v = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, $b = \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix}$, $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

and define a transformation T: $R^2 \rightarrow R^2$ and T(x) = Ax then

(a) find T(v)

(b) find x $\in R^2$ whose image under T is b.

3. Find the LU factorization of

$$\begin{bmatrix} 2 & 4 & -1 & 5 & -2 \\ -4 & -5 & 3 & -8 & 1 \\ 2 & -5 & -4 & 1 & 8 \\ -6 & 0 & 7 & -3 & 1 \end{bmatrix}$$

4. Find a least square solution of the inconsistent system Ax = b for

$A = \begin{bmatrix} 1 & 2 \\ 2 & -3 \\ -1 & 3 \end{bmatrix}$, $b = \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix}$

Group B

Attempt any TEN questions. (10 x 5 = 50)

5. Determine the column of the matrix A are linearly independent, where

$$A = \begin{bmatrix} 0 & 1 & 4 \\ 1 & 2 & -1 \\ 5 & 8 & 0 \end{bmatrix}$$

6. When two column vectors in R^2 are equal? Give an example. Compute u+3v, u-2v where [1+4]

$$u = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}, \quad v = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$$

7. Let $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ **and define T: $R^2 \rightarrow R^2$ by T(x) = Ax, find the image under T of**

$u = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$ **and** $v = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$

8. Find the eigenvalues of

$$A = \begin{bmatrix} 3 & 6 & -8 \\ 0 & 0 & 6 \\ 0 & 0 & 2 \end{bmatrix}$$

9. Define null space of a matrix A. Let

$A = \begin{bmatrix} -1 & -3 & 2 \\ -5 & -9 & 1 \end{bmatrix}$**, and**

Then show that v is in the null A.

10. Verify that 1^k, $(-2)^k$, 3^k are linearly independent signals.

11. If . Find a formula A^n, where A = PDP^{-1} and

and

12. Find a unit vector v of u = (1, -2, 2,3) in the direction of u.

13. Prove that the two vectors u and v are perpendicular to each other if and only if the line through u is a perpendicular bisector of the line segment from -u to v.

14. Let an operation * be defined on Q^+ by a*b = ab/2. Then show that Q^+ forms a group.

15. Define a ring and show that the set of real numbers with respect to addition and multiplication operation is a ring. [2+3]