Tribhuwan University
Institute of Science and Technology
2076

Bachelor Level / second-semester / Science — Full marks: 80
Computer Science and Information Technology(MTH163) — Pass marks: 32
(Mathematics II) — Time: 3 hours
Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

Group A (3 x 10 = 30)

Attempt any THREE questions.

1. When a system of linear equation is consistent and inconsistent? Give an example for each. Test the consistency and solve the system of equations: x - 2y = 5, -x + y + 5z = 2, y + z = 0

2. What is the condition of a matrix to have an inverse? Find the inverse of the matrix $A=\begin{bmatrix}5 & 1 & 2\\ 1 & 0 & 3\\ 4 & -3 & 8\end{bmatrix}$, **If it exists.**

3. Find the least-square solution of Ax=b for $A=\begin{bmatrix}1 & -6\\ 1 & -2\\ 1 & 1\\ 1 & 7\end{bmatrix}$ and $b=\begin{bmatrix}-1\\ 2\\ 1\\ 6\end{bmatrix}$.

4. Let T is a linear transformation. Find the standard matrix of T such that

(i) $T: \mathbb{R}^2 \to \mathbb{R}^4$ **by T(e1) = (3, 1, 3, 1) and** $T(e_2) = (-5, 2, 0, 0)$ **where** $e_1 = (1, 0)$ **and** $e_2 = (0, 1)$;

(ii) $T: \mathbb{R}^2 \to \mathbb{R}^4$ **rotates point as the origin through** $\frac{3\pi}{2}$ **radians counter clockwise.**

(iii) $T: \mathbb{R}^2 \to \mathbb{R}^4$ **Is a vertical shear transformation that maps** e_1 **into** e_1-$2e_2$ **but leaves vector** e_2 **unchanged.**

Group B (10 x 5 = 50)

Attempt any TEN questions.

5. For what value of h will y be in span $\{v_1, v_2, v_3\}$ **if** $v_1=\begin{bmatrix}1\\ -1\\ -2\end{bmatrix}, v_2=\begin{bmatrix}5\\ -4\\ -7\end{bmatrix}, v_3=\begin{bmatrix}-3\\ 1\\ 0\end{bmatrix}$ and $y=\begin{bmatrix}-4\\ 3\\ h\end{bmatrix}$?

6. Let us define a linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ by $T(x)=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} = \begin{bmatrix}-x_2\\ x_1\end{bmatrix}$. **Find the image under** T of $u=\begin{bmatrix}4\\ 1\end{bmatrix}, v=\begin{bmatrix}2\\ 3\end{bmatrix}$, and $u+v=\begin{bmatrix}6\\ 4\end{bmatrix}$.

7. Let $A=\begin{bmatrix}2 & 5\\ -3 & 1\end{bmatrix}$ and $B=\begin{bmatrix}4 & -5\\ 3 & k\end{bmatrix}$. **Determine the value (s) of k if any will make AB = BA.**

8. Define determinant. Compute the determinant without expanding $\begin{vmatrix}-2 & 8 & -9\\ -1 & 7 & 0\\ 1 & -4 & 2\end{vmatrix}$.

9. Define null space . Find the basis for the null space of the matrix $A=\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 4\end{bmatrix}$.

10. Let B = $\{b_1, b_2\}$ **and C =** (c_1, c_2) **be bases for a vector V, and suppose** $b_1 = -c_1 + 4c_2$ **and** $b_2 = 5c_1 - 3c_2$. **Find the change of coordinate matrix for a vector space and find** $[x]_c$ **for** $x = 5b_1 + 3b_2$.

11. Find the eigen values of the matrix $\begin{bmatrix} 6 & 5 \\ -8 & -6 \end{bmatrix}$.

12. Find the QR factorization of the matrix $\begin{bmatrix} 2 & 1 \\ 3 & -1 \end{bmatrix}$.

13. Define binary operation. Determine whether the binary operation * is associative or commutative or both where * is defined on Q by letting $x * y = \frac{x+y}{3}$ **.**

14. Show that the ring is an integral domain.

15. Find the vector x determined by the coordinate vector $[x]_\beta = \begin{bmatrix} -4 \\ 6 \\ -7 \end{bmatrix}$ **where** $\beta = \left\{ \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ -5 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ -7 \\ 3 \end{bmatrix} \right\}$.