Tribhuwan University

Institute of Science and Technology

2077

Bachelor Level / first-semester / Science Full marks: 80 **Computer Science and Information Technology(MTH112)** Pass marks: 32 (Mathematics I (Calculus)) Time: 3 hours Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

Group A(10 x 3 = 30)

Attempt any THREE questions.

1(a) If f(x) = x2 then find $\frac{f(2+h) - f(2)}{h}$ **.**

1(b) Dry air is moving upward. If the ground temperature is 20^0 and the temperature at a height of 1km is 10^0 C, express the temperature T in ^{0}C as a function of the height h (in kilometers), assuming that a linear model is appropriate. (b)Draw the graph of the function in part(a). What does the slope represent? (c) What is the temperature at a height of 2km?(5)

1(c). Find the equation of the tangent to the parabola $y = x^2 + x + 1$ at (0, 1)

2(a)A farmer has 2000 ft of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?[5]

2(b)Sketch the curve[5]

$$y = \frac{1}{x-3}$$

3(a)Show that the converges $\int_1^{\infty} \frac{1}{x^2}$ **and diverges** $\int_1^{\infty} \frac{1}{x}$ **.[2]**
(b) If $f(x, y) = xy/(x^2 + y^2)$, does f(x, y) exist, as (x, y) → (0, 0)?[3]

3(c) A particle moves in a straight line and has acceleration given by $a(t) = 6t^2 + 1$. Its initial velocity is 4m/sec and its initial displacement is s(0) = 5cm. Find its position function s(t).[5]

4. (a) Evaluate[5]

$$\int_3^2 \int_0^{\frac{\pi}{2}} (y + y^2 cosx) dx dy$$

4(b) Find the Maclaurin's series for cos x and prove that it represents cos x for all x.[5]

Group B(10 x 5 = 50)

Attempt any TEN questions.

5. If $f(x) = x^2 - 1$, g(x) = 2x + 1, find fog and gof and domain of fog.

6. Define continuity of a function at a point x = a. Show that the function f(x) = $\sqrt{1-x^2}$ **is continuous on the interval[1, -1].**

7. State Rolle's theorem and verify the Rolle's theorem for $f(x) = x^3 - x^2 - 6x + 2$ in [0, 3]. 8.

Find the third approximation x_3 to the root of the equation $f(x) = x^3 - 2x - 7$, setting x1 = 2. 9.

Find the derivatives of $r(t) = (1 + t^2)i - te^{-t}j + \sin 2tk$ and find the unit tangent vector at t=0.

10. Find the volume of the solid obtained by rotating about the y-axis the region between $y = x$ and $y = x^2$.

11. Solve: $y'' + y' = 0$, $y(0) = 5$, $y(\pi/4) = 3$

12. Show that the series $\sum_{n=0}^{\infty} \frac{1}{1+n^2}$ **converges.**
13. Find a vector perpendicular to the plane that passes through the points:p(1, 4, 6), Q(-2, 5, -1) and R(1. -1,

1) 14. Find the partial derivative of $f(x, y) = x^3 + 2x^3y^3 - 3y^2 + x + y$, at (2,1).

15. Find the local maximum and minimum values, saddle points of $f(x,y) = x^4 + y^4 - 4xy + 1$.